Mark Scheme (Final)
Summer 2007

## GCE

## GCE Mathematics (6663/01)

## General Principal for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

## Method mark for solving 3 term quadratic:

1. Factorisation
$\left(x^{2}+b x+c\right)=(x+p)(x+q)$, where $|p q|=|c|, \quad$ leading to $x=\ldots$
$\left(a x^{2}+b x+c\right)=(m x+p)(n x+q)$, where $|p q|=|c|$ and $|m n|=|a|$, leading to $x=\ldots$
2. Formula

Attempt to use correct formula (with values for $a, b$ and $c$ ).
3. Completing the square

Solving $x^{2}+b x+c=0: \quad(x \pm p)^{2} \pm q \pm c, \quad p \neq 0, q \neq 0, \quad$ leading to $x=\ldots$

## Method marks for differentiation and integration:

## 1. Differentiation

Power of at least one term decreased by 1 . ( $x^{n} \rightarrow x^{n-1}$ )
2. Integration

Power of at least one term increased by 1. $\left(x^{n} \rightarrow x^{n+1}\right)$

## Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

## Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

## Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

## June 2007

## 6663 Core Mathematics C1

Mark Scheme

| Question number | Scheme ${ }^{\text {a }}$ Marks |
| :---: | :---: |
| 1. | $9-5$ or $3^{2}+3 \sqrt{5}-3 \sqrt{5}-\sqrt{5} \times \sqrt{5}$ or $3^{2}-\sqrt{5} \times \sqrt{5} \quad$ or $3^{2}-(\sqrt{5})^{2}$ $=\underline{4}$ |
|  | M1 for an attempt to multiply out. There must be at least 3 correct terms. Allow one sign slip only, no arithmetic errors. <br> e.g. $3^{2}+3 \sqrt{5}-3 \sqrt{5}+(\sqrt{5})^{2}$ is M1A0 <br> $3^{2}+3 \sqrt{5}+3 \sqrt{5}-(\sqrt{5})^{2}$ is M1A0 as indeed is $9 \pm 6 \sqrt{5}-5$ <br> BUT $9+\sqrt{15}-\sqrt{15}-5(=4)$ is M0A0 since there is more than a sign error. <br> $6+3 \sqrt{5}-3 \sqrt{5}-5$ is M0A0 since there is an arithmetic error. <br> If all you see is $9 \pm 5$ that is M1 but please check it has not come from incorrect working. <br> Expansion of $(3+\sqrt{5})(3+\sqrt{5})$ is M0A0 <br> A1cso for 4 only. Please check that no incorrect working is seen. <br> Correct answer only scores both marks. |

\begin{tabular}{|c|c|c|}
\hline Question number \& Scheme \& Marks \\
\hline 2. \& \begin{tabular}{l}
(a) Attempt \(\sqrt[3]{8}\) or \(\sqrt[3]{\left(8^{4}\right)}\)
\[
=\underline{16}
\] \\
(b) \(5 x^{\frac{1}{3}}\) \\
5, \(x^{\frac{1}{3}}\)
\end{tabular} \& \(\begin{array}{ll}\text { M1 } \& \\ \text { A1 } \& \text { (2) } \\ \text { B1, B1 } \& \text { (2) } \\ \& 4\end{array}\) \\
\hline (a)

(b) \& \multicolumn{2}{|l|}{| M1 for: $2\left(\right.$ on its own) or $\left(2^{3}\right)^{\frac{4}{3}}$ or $\sqrt[3]{8}$ or $(\sqrt[3]{8})^{4}$ or $2^{4}$ or $\sqrt[3]{8^{4}}$ or $\sqrt[3]{4096}$ $8^{3}$ or 512 or $(4096)^{\frac{1}{3}}$ is M0 |
| :--- |
| A1 for 16 only |
| $1^{\text {st }}$ B1 for 5 on its own or $\times$ something. |
| So e.g. $\frac{5 x^{\frac{4}{3}}}{x}$ is B1 But $5^{\frac{1}{3}}$ is B0 |
| An expression showing cancelling is not sufficient |
| (see first expression of QC0184500123945 the mark is scored for the second expression) $2^{\text {nd }}$ B1 for $x^{\frac{1}{3}}$ |
| Can use ISW (incorrect subsequent working) |
| e.g $5 x^{\frac{4}{3}}$ scores B1B0 but it may lead to $\sqrt[3]{5 x^{4}}$ which we ignore as ISW. |
| Correct answers only score full marks in both parts. |} <br>

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\end{tabular}



| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 4. | (a) Identify $a=5$ and $d=2$ <br> (May be implied) $\begin{gathered} \left(u_{200}=\right) a+(200-1) d \quad(=5+(200-1) \times 2) \\ =\underline{403}(\mathrm{p}) \text { or }(£) \underline{4.03} \end{gathered}$ <br> (b) $\begin{aligned} \left(S_{200}\right. & =) \frac{200}{2}[2 a+(200-1) d] \text { or } \frac{200}{2}(a+\text { "their } 403 ") \\ & =\frac{200}{2}[2 \times 5+(200-1) \times 2] \text { or } \frac{200}{2}(5+\text { "their } 403 ") \\ & =\underline{40800} \text { or } \underline{£ 408} \end{aligned}$ | B1 <br> M1 <br> A1 (3) <br> M1 <br> A1 <br> A1 (3) |
| (a) | B1 can be implied if the correct answer is obtained. If 403 is not obtained then the values of $a$ and $d$ must be clearly identified as $a=5$ and $d=2$. <br> This mark can be awarded at any point. <br> M1 for attempt to use $n$th term formula with $n=200$. Follow through their $a$ and $d$. <br> Must have use of $n=200$ and one of $a$ or $d$ correct or correct follow through. <br> Must be 199 not 200. |  |
| N.B. | A1 for 403 or 4.03 (i.e. condone missing $£$ sign here). Condone $£ 403$ here $a=3, d=2$ is B 0 and $a+200 d$ is M0 BUT $3+200 \times 2$ is B 1 M 1 and Answer only of 403 (or 4.03 ) scores $3 / 3$. | it leads to 403. |
| (b) | M1 for use of correct sum formula with $n=200$. Follow through their $a$ and $d$ and their 403. <br> Must have some use of $n=200$, and some of $a, d$ or $l$ correct or correct follow through. <br> $1^{\text {st }} \mathrm{A} 1$ for any correct expression (i.e. must have $a=5$ and $d=2$ ) but can f.t. their 403 still. $2^{\text {nd }} \mathrm{A} 1$ for 40800 or $£ 408$ (i.e. the $£$ sign is required before we accept 408 this time). 40800 p is fine for A1 but $£ 40800$ is A0. |  |
| ALT | $\underline{\text { Listing }}$ |  |
| (a) (b) | $\sum_{r=1}^{200}(2 r+3)$. Give M1 for $2 \times \frac{200}{2} \times(201)+3 k($ with $k>1), \mathrm{A} 1$ for $k=200$ and A1 for 40800. |  |

\begin{tabular}{|c|c|}
\hline Question number \& Scheme ${ }^{\text {a }}$ Marks <br>
\hline 5.

S.C. \& | (a) |
| :--- |
| Translation parallel to $x$-axis |
| Top branch intersects $+\mathrm{ve} y$-axis |
| Lower branch has no intersections |
| No obvious overlap $\qquad$ $\begin{equation*} \left(0, \frac{3}{2}\right) \text { or } \frac{3}{2} \text { marked on } y \text {-axis } \tag{3} \end{equation*}$ |
| (b) $x=-2, \quad y=0$ |
| [Allow ft on first B 1 for $x=2$ when translated "the wrong way" but must be compatible with their sketch.] | <br>

\hline (a)

(b)

S.C. \& | M1 for a horizontal translation - two branches with one branch cutting $y$-axis only. |
| :--- |
| If one of the branches cuts both axes (translation up and across) this is M0. |
| A1 for a horizontal translation to left. Ignore any figures on axes for this mark. |
| B1 for correct intersection on positive $y$-axis. More than 1 intersection is B0. |
| $x=0$ and $y=1.5$ in a table alone is insufficient unless intersection of their sketch is with $+\mathrm{ve} y$-axis. |
| A point marked on the graph overrides a point given elsewhere. |
| $1^{\text {st }} \mathrm{B} 1$ for $x=-2$. NB $x \neq-2$ is B 0 . |
| Can accept $x=+2$ if this is compatible with their sketch. |
| Usually they will have M1A0 in part (a) (and usually B0 too) |
| $2^{\text {nd }}$ B1 for $y=0$. |
| If $x=-2$ and $y=0$ and some other asymptotes are also given award B1B0 |
| The asymptote equations should be clearly stated in part (b). Simply marking $x=-2$ or $y=0$ on the sketch is insufficient unless they are clearly marked "asymptote $x=-2$ " etc. | <br>

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\end{tabular}

| Question number | Scheme ${ }^{\text {a }}$ Marks |
| :---: | :---: |
| 6. | (a) $\begin{gathered}2 x^{2}-x(x-4)=8 \\ x^{2}+4 x-8=0\end{gathered}$ <br> (b) $x=\frac{-4 \pm \sqrt{4^{2}-(4 \times 1 \times-8)}}{2}$ <br> or $(x+2)^{2} \pm 4-8=0$ <br> $x=-2 \pm$ (any correct expression) <br> or $\quad \sqrt{12}=\sqrt{4} \sqrt{3}=2 \sqrt{3}$ <br> M : Attempt at least one $y$ value $\underline{x=-2-2 \sqrt{3}, \quad y=-6-2 \sqrt{3}}$ |
| (a) (b) | M1 for correct attempt to form an equation in $x$ only. Condone sign errors/slips but attempt at this line must be seen. E.g. $2 x^{2}-x^{2} \pm 4 x=8$ is OK for M1. <br> A1cso for correctly simplifying to printed form. No incorrect working seen. The $=0$ is required. <br> These two marks can be scored in part (b). For multiple attempts pick best. <br> $1^{\text {st }} \mathrm{M} 1$ for use of correct formula. If formula is not quoted then a fully correct substitution is required. Condone missing $x=$ or just + or - instead of $\pm$ for M1. <br> For completing the square must have as printed or better. <br> If they have $x^{2}-4 x-8=0$ then M1 can be given for $(x-2)^{2} \pm 4-8=0$. <br> $1^{\text {st }} \mathrm{A} 1$ for $-2 \pm$ any correct expression. (The $\pm$ is required but $x=$ is not) <br> B1 for simplifying the surd e.g. $\sqrt{48}=4 \sqrt{3}$. Must reduce to $b \sqrt{3}$ so $\sqrt{16} \sqrt{3}$ or $\sqrt{4} \sqrt{3}$ are OK. <br> $2^{\text {nd }} \mathrm{M} 1$ for attempting to find at least one $y$ value. Substitution into one of the given equations and an attempt to solve for $y$. <br> $2^{\text {nd }} \mathrm{A} 1$ for correct $y$ answers. Pairings need not be explicit but they must say which is $x$ and which $y$. Mis-labelling $x$ and $y$ loses final A1 only. |



| Question number | Scheme Marks |
| :---: | :---: |
| 8. | (a) $\left(a_{2}=\right) 3 k+5 \quad$ [must be seen in part (a) or labelled $a_{2}=$ ] <br> (b) $\left(a_{3}=\right) 3(3 k+5)+5$ $\begin{equation*} =9 k+20 \tag{*} \end{equation*}$ <br> (c)(i) $a_{4}=3(9 k+20)+5 \quad(=27 k+65)$ $\sum_{r=1}^{4} a_{r}=k+(3 k+5)+(9 k+20)+(27 k+65)$ <br> (ii) $=40 k+90$ $=\underline{10(4 k+9)} \quad(\text { or explain why divisible by } 10)$ |
| (b) | M1 for attempting to find $a_{3}$, follow through their $a_{2} \neq k$. <br> A1cso for simplifying to printed result with no incorrect working seen. <br> $1^{\text {st }}$ M1 for attempting to find $a_{4}$. Can allow a slip here e.g. $3(9 k+20)$ [i.e. forgot +5 ] <br> $2^{\text {nd }}$ M1 for attempting sum of 4 relevant terms, follow through their (a) and (b). <br> Must have 4 terms starting with $k$. <br> Use of arithmetic series formulae at this point is MOA0A0 <br> $1^{\text {st }} \mathrm{A} 1 \quad$ for simplifying to $40 k+90$ or better <br> $2^{\text {nd }}$ A1ft for taking out a factor of 10 or dividing by 10 or an explanation in words true $\forall k$. <br> Follow through their sum of 4 terms provided that both Ms are <br> scored and their sum is divisible by 10 . <br> A comment is not required. <br> e.g. $\frac{40 k+90}{10}=4 k+9$ is OK for this final A1. $\sum_{r=2}^{5} a_{r}=120 k+290=10(12 k+29) \text { can have M1M0A0A1 ft. }$ |

\begin{tabular}{|c|c|}
\hline Question number \& Scheme \({ }^{\text {a }}\) Marks \\
\hline 9. \& \begin{tabular}{l}
(a) \\
\(\mathrm{f}(x)=\frac{6 x^{3}}{3}-\frac{10 x^{2}}{2}-12 x(+C)\)
\[
x=5: \quad 250-125-60+C=65 \quad C=0
\] \\
(b) \(x\left(2 x^{2}-5 x-12\right)\) or \(\left(2 x^{2}+3 x\right)(x-4)\) or \((2 x+3)\left(x^{2}-4 x\right)\)
\[
=x(2 x+3)(x-4)
\] \\
(c)
\end{tabular} \\
\hline (a)
(b)

(b) \& | $1^{\text {st }}$ M1 for attempting to integrate, $x^{n} \rightarrow x^{n+1}$ |
| :--- |
| $1^{\text {st }} \mathrm{A} 1$ for all $x$ terms correct, need not be simplified. Ignore $+C$ here. |
| $2^{\text {nd }} \mathrm{M} 1$ for some use of $x=5$ and $\mathrm{f}(5)=65$ to form an equation in $C$ based on their integration. |
| There must be some visible attempt to use $x=5$ and $\mathrm{f}(5)=65$. No $+C$ is M0. |
| $2^{\text {nd }} \mathrm{A} 1$ for $C=0$. This mark cannot be scored unless a suitable equation is seen. |
| M1 for attempting to take out a correct factor or to verify. Allow usual errors on signs. |
| They must get to the equivalent of one of the given partially factorised expressions or, if verifying, $x\left(2 x^{2}+3 x-8 x-12\right)$ i.e. with no errors in signs. |
| A1cso for proceeding to printed answer with no incorrect working seen. Comment not required. |
| This mark is dependent upon a fully correct solution to part (a) so M1A1M0A0M1A0 for (a) \& (b). Will be common or M1A1M1A0M1A0. To score 2 in (b) they must score 4 in (a). |
| $1^{\text {st }} \mathrm{B} 1$ for positive $x^{3}$ shaped curve (with a max and a min) positioned anywhere. |
| $2^{\text {nd }}$ B1 for any curve that passes through the origin (B0 if it only touches at the origin) |
| $3^{\text {rd }} \mathrm{B} 1$ for the two points clearly given as coords or values marked in appropriate places on $x$ axis. Ignore any extra crossing points (they should have lost first B1). |
| Condone $(1.5,0)$ if clearly marked on -ve $x$-axis. Condone $(0,4)$ etc if marked on $+\mathrm{ve} x$ axis. Curve can stop (i.e. not pass through) at $(-1.5,0)$ and $(4,0)$. |
| A point on the graph overrides coordinates given elsewhere. | <br>

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\end{tabular}



\begin{tabular}{|c|c|}
\hline Question number \& Scheme \({ }^{\text {a }}\) Marks \\
\hline 11. \& \begin{tabular}{l}
(a) \(y=-\frac{3}{2} x(+4)\) Gradient \(=-\frac{3}{2}\) \\
(b)
\[
\begin{aligned}
\& 3 x+2=-\frac{3}{2} x+4 \quad x=\ldots, \frac{4}{9} \\
\& y=3\left(\frac{4}{9}\right)+2=\frac{10}{3}\left(=3 \frac{1}{3}\right)
\end{aligned}
\] \\
(c) Where \(y=1, \quad l_{1}: x_{A}=-\frac{1}{3} \quad l_{2}: x_{B}=2 \quad \mathrm{M}\) : Attempt one of these
\[
\begin{aligned}
\text { Area } \& =\frac{1}{2}\left(x_{B}-x_{A}\right)\left(y_{P}-1\right) \\
\& =\frac{1}{2} \times \frac{7}{3} \times \frac{7}{3}=\frac{49}{18}=2 \frac{13}{18}
\end{aligned}
\]
\begin{tabular}{l|lr} 
M1 A1 \& (2) \\
M1, A1 \& \\
A1 \& (3) \\
M1 A1 \& \\
M1 \& \\
A1 \& (4) \\
\& 9
\end{tabular}
\end{tabular} \\
\hline (a)
(b)

(c) \& | M1 for an attempt to write $3 x+2 y-8=0$ in the form $y=m x+c$ |
| :--- |
| or a full method that leads to $m=$, e.g find 2 points, and attempt gradient using $\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$ |
| e.g. finding $y=-1.5 x+4$ alone can score M1 (even if they go on to say $m=4$ ) |
| A1 for $m=-\frac{3}{2}($ can ignore the $+c)$ or $\frac{\mathrm{d} y}{\mathrm{~d} x}=-\frac{3}{2}$ |
| M1 for forming a suitable equation in one variable and attempting to solve leading to $x=$..or $y=$ |
| $1^{\text {st }}$ A1 for any exact correct value for $x$ |
| $2^{\text {nd }} \mathrm{A} 1$ for any exact correct value for $y$ |
| (These 3 marks can be scored anywhere, they may treat (a) and (b) as a single part) |
| $1^{\text {st }}$ M1 for attempting the $x$ coordinate of $A$ or $B$. One correct value seen scores M1. |
| $1^{\text {st }} \mathrm{A} 1$ for $x_{A}=-\frac{1}{3}$ and $x_{B}=2$ |
| $2^{\text {nd }}$ M1 for a full method for the area of the triangle - follow through their $x_{A}, x_{B}, y_{P}$. |
| e.g. determinant approach $\frac{1}{2}\left\|\begin{array}{cccc}2 & -\frac{1}{3} & \frac{4}{9} & 2 \\ 1 & 1 & \frac{10}{3} & 1\end{array}\right\|=\frac{1}{2}\left\|2-\ldots-\left(-\frac{1}{3} \ldots\right)\right\|$ |
| $2^{\text {nd }}$ A1 for $\frac{49}{18}$ or an exact equivalent. |
| All accuracy marks require answers as single fractions or mixed numbers not necessarily in lowest terms. | <br>

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