## Mark Scheme (Final)

Summer 2007

## GCE

## GCE Mathematics (6664/01)

## General Principal for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

## Method mark for solving 3 term quadratic:

1. Factorisation
$\left(x^{2}+b x+c\right)=(x+p)(x+q)$, where $|p q|=|c|, \quad$ leading to $x=\ldots$
$\left(a x^{2}+b x+c\right)=(m x+p)(n x+q)$, where $|p q|=|c|$ and $|m n|=|a|$, leading to $x=\ldots$
2. Formula

Attempt to use correct formula (with values for $a, b$ and $c$ ).
3. Completing the square

Solving $x^{2}+b x+c=0: \quad(x \pm p)^{2} \pm q \pm c, p \neq 0, q \neq 0, \quad$ leading to $x=\ldots$

## Method marks for differentiation and integration:

## 1. Differentiation

Power of at least one term decreased by 1 . ( $x^{n} \rightarrow x^{n-1}$ )
2. Integration

Power of at least one term increased by 1. $\left(x^{n} \rightarrow x^{n+1}\right)$

## Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

## Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

## Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

## June 2007

6664 Core Mathematics C2
Mark Scheme

| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 1. | $\begin{aligned} & \int x^{-\frac{1}{2}} \mathrm{~d} x=\frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} \quad \text { (Or equivalent, such as } 2 x^{\frac{1}{2}} \text {, or } 2 \sqrt{x} \text { ) } \\ & {\left[\frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}\right]_{1}^{8}=2 \sqrt{ } 8-2=-2+4 \sqrt{ } 2 \quad[\text { or } 4 \sqrt{ } 2-2 \text {, or } 2(2 \sqrt{ } 2-1) \text {, or } 2(-1+2 \sqrt{ } 2)]} \end{aligned}$ | M1 A1 <br> M1 A1 <br> (4) $4$ |
|  | $1^{\text {st }} \mathrm{M}: x^{-\frac{1}{2}} \rightarrow k x^{\frac{1}{2}}, k \neq 0$ <br> $2^{\text {nd }} \mathrm{M}$ : Substituting limits 8 and 1 into a 'changed' function (i.e. not $\frac{1}{\sqrt{x}}$ or $x^{-\frac{1}{2}}$ ), and subtracting, either way round. <br> $2^{\text {nd }} A$ : This final mark is still scored if $-2+4 \sqrt{ } 2$ is reached via a decimal. <br> N.B. Integration constant $+C$ may appear, e.g. $\left[\frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}+C\right]_{1}^{8}=(2 \sqrt{ } 8+C)-(2+C)=-2+4 \sqrt{ } 2$ <br> (Still full marks) <br> But... a final answer such as $-2+4 \sqrt{ } 2+C$ is A0. <br> N.B. It will sometimes be necessary to 'ignore subsequent working' (isw) after a correct form is seen, e.g. $\int x^{-\frac{1}{2}} \mathrm{~d} x=\frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}$ (M1 A1), followed by incorrect simplification $\int x^{-\frac{1}{2}} \mathrm{~d} x=\frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}=\frac{1}{2} x^{\frac{1}{2}}$ (still M1 A1).... The second M mark is still available for substituting 8 and 1 into $\frac{1}{2} x^{\frac{1}{2}}$ and subtracting. |  |


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| :---: | :---: | :---: |
| 2. | (a) $f(2)=24-20-32+12=-16$ <br> (M: Attempt $\mathrm{f}(2)$ or $\mathrm{f}(-2)$ ) <br> (If continues to say 'remainder $=16$ ', isw) <br> Answer must be seen in part (a), not part (b). <br> (b) $\begin{aligned} & (x+2)\left(3 x^{2}-11 x+6\right) \\ & (x+2)(3 x-2)(x-3) \end{aligned}$ <br> (If continues to 'solve an equation', isw) | M1 A1 <br> M1 A1 <br> M1 A1 <br> (4) |
|  | (a) Answer only (if correct) scores both marks. (16 as 'answer only' is M0 A0). <br> Alternative (long division): <br> Divide by $(x-2)$ to get $\left(3 x^{2}+a x+b\right), a \neq 0, b \neq 0$. [M1] <br> $\left(3 x^{2}+x-14\right)$, and -16 seen. <br> (If continues to say 'remainder $=16$ ', isw) <br> (b) First M requires division by $(x+2)$ to get $\left(3 x^{2}+a x+b\right), a \neq 0, b \neq 0$. <br> Second M for attempt to factorise their quadratic, even if wrongly obtained, perhaps with a remainder from their division. <br> Usual rule: $\left(k x^{2}+a x+b\right)=(p x+c)(q x+d)$, where $\|p q\|=\|k\|$ and $\|c d\|=\|b\|$. Just solving their quadratic by the formula is M0. <br> "Combining" all 3 factors is not required. <br> Alternative (first 2 marks): <br> $(x+2)\left(3 x^{2}+a x+b\right)=3 x^{3}+(6+a) x^{2}+(2 a+b) x+2 b=0$, then compare coefficients to find values of $a$ and $b$. [M1] $\overline{a=-11}, b=6 \quad[\mathrm{~A} 1]$ <br> Alternative: <br> Factor theorem: Finding that $\mathrm{f}(3)=0 \therefore$ factor is, $\quad(x-3) \quad[\mathrm{M} 1, \mathrm{~A} 1]$ <br> Finding that $\mathrm{f}\left(\frac{2}{3}\right)=0 \therefore$ factor is, $\quad(3 x-2) \quad$ [M1, A1] <br> If just one of these is found, score the first 2 marks M1 A1 M0 A0. <br> Losing a factor of 3: $(x+2)\left(x-\frac{2}{3}\right)(x-3)$ scores M1 A1 M1 A0. <br> Answer only, one sign wrong: e.g. $(x+2)(3 x-2)(x+3)$ scores M1 A1 M1 A0. |  |


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| :---: | :---: | :---: |
| 3. | (a) $1+6 k x \quad$ [Allow unsimplified versions, e.g. $\left.1^{6}+6\left(1^{5}\right) k x,{ }^{6} C_{0}+{ }^{6} C_{1} k x\right]$ $+\frac{6 \times 5}{2}(k x)^{2}+\frac{6 \times 5 \times 4}{3 \times 2}(k x)^{3} \quad$ [See below for acceptable versions] <br> N.B. THIS NEED NOT BE SIMPLIFIED FOR THE A1 (isw is applied) <br> (b) $6 k=15 k^{2} \quad k=\frac{2}{5}$ (or equiv. fraction, or 0.4) $\quad$ (Ignore $k=0$, if seen) <br> (c) $c=\frac{6 \times 5 \times 4}{3 \times 2}\left(\frac{2}{5}\right)^{3}=\frac{32}{25} \quad$ (or equiv. fraction, or 1.28) <br> (Ignore $x^{3}$, so $\frac{32}{25} x^{3}$ is fine) | B1 <br> M1 A1 <br> M1 A1cso <br> (2) <br> A1cso <br> (1) |
|  | (a) The terms can be 'listed' rather than added. <br> M1: Requires correct structure: 'binomial coefficients' (perhaps from Pascal's triangle), increasing powers of $x$. Allow a 'slip' or 'slips' such as: $\begin{array}{ll} +\frac{6 \times 5}{2} k x^{2}+\frac{6 \times 5 \times 4}{3 \times 2} k x^{3}, & +\frac{6 \times 5}{2}(k x)^{2}+\frac{6 \times 5}{3 \times 2}(k x)^{3} \\ +\frac{5 \times 4}{2} k x^{2}+\frac{5 \times 4 \times 3}{3 \times 2} k x^{3}, & +\frac{6 \times 5}{2} x^{2}+\frac{6 \times 5 \times 4}{3 \times 2} x^{3} \end{array}$ <br> But: $15+k^{2} x^{2}+20+k^{3} x^{3}$ or similar is M0. <br> Both $x^{2}$ and $x^{3}$ terms must be seen. <br> $\binom{6}{2}$ and $\binom{6}{3}$ or equivalent such as ${ }^{6} C_{2}$ and ${ }^{6} C_{3}$ are acceptable, and <br> even $\left(\frac{6}{2}\right)$ and $\left(\frac{6}{3}\right)$ are acceptable for the method mark. <br> A1: Any correct (possibly unsimplified) version of these 2 terms. <br> $\binom{6}{2}$ and $\binom{6}{3}$ or equivalent such as ${ }^{6} C_{2}$ and ${ }^{6} C_{3}$ are acceptable. <br> Descending powers of $x$ : <br> Can score the M mark if the required first 4 terms are not seen. <br> Multiplying out $(1+k x)(1+k x)(1+k x)(1+k x)(1+k x)(1+k x)$ : <br> M1: A full attempt to multiply out (power 6) <br> B1 and A1 as on the main scheme. <br> (b) M: Equating the coefficients of $x$ and $x^{2}$ (even if trivial, e.g. $6 k=15 k$ ). <br> Allow this mark also for the 'misread': equating the coefficients of $x^{2}$ and $x^{3}$. An equation in $k$ alone is required for this M mark, although... <br> ...condone $6 k x=15 k^{2} x^{2} \Rightarrow\left(6 k=15 k^{2} \Rightarrow\right) k=\frac{2}{5}$. |  |


| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 4. | (a) $\begin{align*} & 4^{2}=5^{2}+6^{2}-(2 \times 5 \times 6 \cos \theta) \\ & \cos \theta=\frac{5^{2}+6^{2}-4^{2}}{2 \times 5 \times 6} \\ & \quad\left(=\frac{45}{60}\right)=\frac{3}{4} \tag{*} \end{align*}$ <br> (b) $\sin ^{2} A+\left(\frac{3}{4}\right)^{2}=1$ <br> (or equiv. Pythag. method) <br> $\left(\sin ^{2} A=\frac{7}{16}\right) \quad \sin A=\frac{1}{4} \sqrt{ } 7 \quad$ or equivalent exact form, e.g. $\sqrt{\frac{7}{16}}, \sqrt{0.4375}$ | M1 <br> A1 <br> A1cso <br> (3) <br> M1 <br> A1 <br> (2) |
|  | (a) M: Is also scored for $5^{2}=4^{2}+6^{2}-(2 \times 4 \times 6 \cos \theta)$ or $\quad 6^{2}=5^{2}+4^{2}-(2 \times 5 \times 4 \cos \theta)$ $\text { or } \cos \theta=\frac{4^{2}+6^{2}-5^{2}}{2 \times 4 \times 6} \text { or } \cos \theta=\frac{5^{2}+4^{2}-6^{2}}{2 \times 5 \times 4} .$ <br> $1^{\text {st }} \mathrm{A}$ : Rearranged correctly and numerically correct (possibly unsimplified), in the form $\cos \theta=\ldots$ or $60 \cos \theta=45$ (or equiv. in the form $p \cos \theta=q$ ). <br> Alternative (verification): $\begin{equation*} 4^{2}=5^{2}+6^{2}-\left(2 \times 5 \times 6 \times \frac{3}{4}\right) \tag{M1} \end{equation*}$ <br> Evaluate correctly, at least to $16=25+36-45$ [A1] <br> Conclusion (perhaps as simple as a tick). <br> [A1cso] (Just achieving $16=16$ is insufficient without at least a tick). <br> (b) M: Using a correct method to find an equation in $\sin ^{2} A$ or $\sin A$ which would give an exact value. <br> Correct answer without working (or with unclear working or decimals): Still scores both marks. |  |


| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 5. | (a) 1.414 (allow also exact answer $\sqrt{ } 2$ ), <br> 3.137 <br> Allow awrt <br> (b) $\frac{1}{2}(0.5) \ldots$ $\ldots .\{0+6+2(0.530+1.414+3.137)\}$ <br> $=4.04 \quad$ (Must be 3 s.f.) <br> (c) Area of triangle $=\frac{1}{2}(2 \times 6)$ <br> (Could also be found by integration, or even by the trapezium rule on $y=3 x$ ) <br> Area required = Area of triangle - Answer to (b) (Subtract either way round) <br> $6-4.04=1.96$ <br> Allow awrt <br> (ft from (b), dependent on the B1, and on answer to (b) less than 6) | $\mathrm{B} 1, \mathrm{~B} 1$ <br> B1 <br> M1 A1ft <br> A1 <br> (4) <br> B1 <br> M1 <br> A1ft <br> (3) |
|  | (a) If answers are given to only 2 d.p. (1.41 and 3.14), this is B0 B0, but full mark\$ can be given in part (b) if 4.04 is achieved. <br> (b) Bracketing mistake: i.e. $\frac{1}{2}(0.5)(0+6)+2(0.530+1.414+3.137)$ <br> scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given). <br> Alternative (finding and adding separate areas): $\frac{1}{2} \times \frac{1}{2}$ (Triangle/trapezium formulae, and height of triangle/trapezium) [B1] Fully correct method for total area, with values from table. <br> [M1, A1ft] 4.04 <br> (c) B1: Can be given for 6 with no working, but should not be given for 6 obtained from wrong working. <br> A1ft: This is a dependent follow-through: the B1 for 6 must have been scored, and the answer to (b) must be less than 6 . |  |


| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 6. | (a) $x=\frac{\log 0.8}{\log 8}$ or $\log _{8} 0.8, \quad=-0.107 \quad$ Allow awrt <br> (b) $2 \log x=\log x^{2}$ $\log x^{2}-\log 7 x=\log \frac{x^{2}}{7 x}$ <br> "Remove logs" to form equation in $x$, using the base correctly: $\frac{x^{2}}{7 x}=3$ $x=21 \quad \text { (Ignore } x=0, \text { if seen) }$ | M1, A1  <br> B1  <br> M1  <br> M1  <br> A1cso  <br>   |
|  | (a) Allow also the 'implicit' answer $8^{-0.107}$ (M1 A1). <br> Answer only: -0.107 or awrt: Full marks. <br> Answer only: -0.11 or awrt (insufficient accuracy): M1 A0 <br> Trial and improvement: Award marks as for "answer only". <br> (b) Alternative: <br> Alternative: $\begin{array}{lll} \hline \log 7 x=\log 7+\log x & & \text { B1 } \\ 2 \log x-(\log 7+\log x)=1 & & \\ \log _{3} x=1+\log _{3} 7 & & \\ x=3^{\left(1+\log _{3} 7\right)} \quad\left(=3^{2.771 \ldots}\right) & \text { or } & \log _{3} x=\log _{3} 3+\log _{3} 7 \\ x=21 & & \text { M1 } \\ x= & & \end{array}$ <br> Attempts using change of base will usually require the same steps as in the main scheme or alternatives, so can be marked equivalently. <br> A common mistake: <br> $\log x^{2}-\log 7 x=\frac{\log x^{2}}{\log 7 x}$ <br> B1 M0 <br> $\frac{x^{2}}{7 x}=3 \quad x=21$ <br> M1('Recovery'), but A0 |  |


| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 7. | (a) Gradient of $A M: \quad \frac{1-(-2)}{3-1}=\frac{3}{2} \quad$ or $\frac{-3}{-2}$ <br> Gradient of $l:=-\frac{2}{3} \quad$ M: use of $m_{1} m_{2}=-1$, or equiv. <br> $y-1=-\frac{2}{3}(x-3) \quad$ or $\frac{y-1}{x-3}=-\frac{2}{3} \quad[3 y=-2 x+9] \quad$ (Any equiv. form) <br> (b) $x=6: \quad 3 y=-12+9=-3 \quad y=-1 \quad($ or show that for $y=-1, x=6) \quad(*)$ <br> (A conclusion is not required). <br> (c) $\left(r^{2}=\right)(6-1)^{2}+(-1-(-2))^{2}$ <br> M: Attempt $r^{2}$ or $r$ <br> N.B. Simplification is not required to score M1 A1 <br> $(x \pm 6)^{2}+(y \pm 1)^{2}=k, \quad k \neq 0 \quad\left(\right.$ Value for $k$ not needed, could be $r^{2}$ or $\left.r\right)$ $(x-6)^{2}+(y+1)^{2}=26$ (or equiv.) <br> Allow $(\sqrt{26})^{2}$ or other exact equivalents for 26 . <br> (But... $(x-6)^{2}+(y--1)^{2}=26$ scores M1 A0) <br> (Correct answer with no working scores full marks) | B1 <br> M1 <br> M1 A1 <br> B1 <br> M1 A1 <br> M1 <br> A1 <br> (4) |
|  | (a) $2^{\text {nd }}$ M1: eqn. of a straight line through $(3,1)$ with any gradient except 0 or $\infty$. <br> Alternative: Using $(3,1)$ in $y=m x+c$ to find a value of $c$ scores M1, but an equation (general or specific) must be seen. <br> Having coords the wrong way round, e.g. $y-3=-\frac{2}{3}(x-1)$, loses the $2^{\text {nd }} \mathrm{M}$ mark unless a correct general formula is seen, e.g. $y-y_{1}=m\left(x-x_{1}\right)$. <br> If the point $P(6,-1)$ is used to find the gradient of $M P$, maximum marks are (a) B0 M0 M1 A1 (b) B0. <br> (c) $1^{\text {st }} \mathrm{M} 1$ : Condone one slip, numerical or sign, inside a bracket. <br> Must be attempting to use points $P(6,-1)$ and $A(1,-2)$, or perhaps $P$ and $B$. (Correct coordinates for $B$ are $(5,4)$ ). <br> $1^{\text {st }} \mathrm{M}$ alternative is to use a complete Pythag. method on triangle MAP, n.b. $M P=M A=\sqrt{13}$. <br> Special case: <br> If candidate persists in using their value for the $y$-coordinate of $P$ instead of the given -1 , allow the M marks in part (c) if earned. |  |



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| :---: | :---: | :---: |
| 9. | (a) <br> Sine wave (anywhere) with at least 2 turning points. <br> Starting on positive $y$-axis, going up to a max., then min. below $x$-axis, no further turning points in range, finishing above $x$-axis at $x=2 \pi$ or $360^{\circ}$. There must be some indication of scale on the $y$-axis... (e.g. 1, -1 or 0.5 ) <br> Ignore parts of graph outside 0 to $2 \pi$. <br> n.b. Give credit if necessary for what is seen on an initial sketch (before any transformation has been performed). <br> (b) $\left(0, \frac{1}{2}\right),\left(\frac{5 \pi}{6}, 0\right),\left(\frac{11 \pi}{6}, 0\right)$ <br> (Ignore any extra solutions) <br> (Not $\left.150^{\circ}, 330^{\circ}\right)$ $\left(\pi-\frac{\pi}{6}\right)$ and $\left(2 \pi-\frac{\pi}{6}\right)$ are insufficient, but if both are seen allow B1 B0. <br> (c) awrt 0.71 radians $(0.70758 \ldots)$ or $\operatorname{awrt} 40.5^{\circ}(40.5416 \ldots) \quad(\alpha)$ $(\pi-\alpha) \quad(2.43 \ldots)$ or $(180-\alpha) \underline{\text { if } \alpha \text { is in degrees. }} \quad\left[\underline{\text { NOT }} \pi-\left(\alpha-\frac{\pi}{6}\right)\right]$ <br> Subtract $\frac{\pi}{6}$ from $\alpha($ or from $(\pi-\alpha)) \ldots$ or subtract $30 \underline{\text { if } \alpha \text { is in degrees }}$ 0.18 (or $0.06 \pi$ ), <br> 1.91 (or $0.61 \pi$ ) <br> Allow awrt <br> (The $1^{\text {st }} \mathrm{A}$ mark is dependent on just the $2^{\text {nd }} \mathrm{M}$ mark) <br> (b) The zeros are not required, i.e. allow 0.5, etc. (and also allow coordinates the wrong way round). <br> These marks are also awarded if the exact intercept values are seen in part (a), but if values in (b) and (a) are contradictory, (b) takes precedence. <br> (c) B1: If the required value of $\alpha$ is not seen, this mark can be given by implication if a final answer rounding to 0.18 or 0.19 (or a final answer rounding to 1.91 or 1.90 ) is achieved. (Also see premature approx. note*). <br> Special case: $\sin \left(x+\frac{\pi}{6}\right)=0.65 \Rightarrow \sin x+\sin \frac{\pi}{6}=0.65 \Rightarrow \sin x=0.15$ $x=\arcsin 0.15=0.15056 \ldots$ and $x=\pi-0.15056=2.99$ (B0 M1 M0 A0 A0) (This special case mark is also available for degrees... $180-8.62 \ldots$ ) <br> Extra solutions outside 0 to $2 \pi$ : Ignore. <br> Extra solutions between 0 and $2 \pi$ : Loses the final A mark. <br> *Premature approximation in part (c): <br> e.g. $\alpha=41^{\circ}, 180-41=139,41-30=11$ and $139-30=109$ <br> Changing to radians: 0.19 and 1.90 <br> This would score B1 (required value of $\alpha$ not seen, but there is a final answer 0.19 (or 1.90 ), M1 M1 A0 A0. | M1 <br> A1 <br> (2) <br> B1, B1, B1 <br> (3) <br> B1 <br> M1 <br> M1 <br> A1, A1 <br> (5) |



