

# ST ALBANS SCHOOL

## 13+ Examination 2016 **MATHEMATICS** (90 minutes)

This is a non-calculator paper.  
There are 14 pages of questions.  
Marks may be given for correct working even if the final answer is incorrect.  
The mark for each question is shown in square brackets.

Surname	First Name

Date of Birth

Score Total



# 13+ MATHEMATICS EXAMINATION

## INSTRUCTIONS

This is a non-calculator paper. There are 14 pages of questions.

The number in the square brackets shows the marks available for each question

**Remember that you should show your methods, as marks may be given for correct working even if the final answer is incorrect.**

$$1) \frac{3}{4} + 1\frac{1}{2} + \frac{1}{3} = 2\frac{7}{12}$$

$$\frac{3}{4} + \frac{1 \times 2 + 1}{2} + \frac{1}{3} \equiv \frac{3}{4} + \frac{3}{2} + \frac{1}{3}$$
$$= \frac{9 + 18 + 4}{12} = \frac{31}{12}$$

$$= 2\frac{7}{12}$$

[2]

$$2) 2\frac{2}{3} \div 2\frac{2}{9} = 1\frac{1}{5}$$

$$\frac{2 \times 3 + 2}{3} \div \frac{2 \times 9 + 2}{9}$$

$$= \frac{8}{3} \div \frac{20}{9}$$

$$= \frac{8}{3} \times \frac{9}{20}$$

$$= \frac{8 \times 3}{3 \times 5} = \frac{8}{5}$$

$$= 1\frac{3}{5}$$

[3]

Page total  $\frac{5}{5}$

3) Use the fact that  $39 \times 32 = 1248$  to write down each of the following:

a)  $13 \times 32 = 416$

$$13 \times 32 = \frac{1248}{3} = 416$$

b)  $3.9 \times 1.6 = 6.24$

$$3.9 \times 1.6 = \frac{1248}{2} = 6.24$$

[2]

c)  $12.48 \div 0.39 = 32$

$$\frac{12.48}{0.39} = \frac{1248}{39} = 32$$

[2]

d)  $39 \times 40 - 39 \times 8 = 1248$

$$39(40-8) = 39 \times 32 \\ = 1248$$

[2]

[2]

Page total  $\frac{-}{8}$

4)

a) Write 1225 as a product of prime factors

$$1225 = 5 \times 5 \times 7 \times 7 \\ = 5^2 \times 7^2$$

5	1225
5	245
7	49
7	7
	1

[2]

b) Use your answer to (a) to find  $\sqrt{1225}$

$$\sqrt{1225} = \sqrt{5^2 \times 7^2} \\ = 5 \times 7 \\ = 35$$

[2]

Page total  $\frac{\quad}{4}$

5)

a) Increase £120 by 35%

$$\begin{aligned} &= € 120 + 35\% \text{ of } 120 \\ &= 120 + \frac{35}{100} \times 120 \\ &= 120 + 42 \\ &= €162 \end{aligned}$$

[2]

b) Shares originally costing £1500 lose 20% of their value and then gain 10%. What is the final value?

loss:

$$\begin{aligned} &= €1500 - 20\% \text{ of } 1500 \\ &= 1500 - \frac{20}{100} \times 1500 \\ &= 1500 - 300 \\ &= €1200 \end{aligned}$$

Gain:

$$\begin{aligned} &€1200 + 10\% \text{ of } 1200 \\ &= 1200 + \frac{10}{100} \times 1200 \\ &= 1200 + 120 \\ &= €1320 \end{aligned}$$

[2]

Page total  $\frac{-}{4}$

6) Simplify the following:

$$\begin{aligned} \text{a) } 5x+3y-2x-y &= 3x+2y \\ &= 5x-2x+3y-y \\ &= 3x+2y \end{aligned}$$

[2]

$$\text{b) } 2a^2+3a^2=5a^2$$

[1]

$$\text{c) } 4a^2b \times 3ab^3 = 12a^3b^4 \quad a^m \times a^n = a^{m+n}$$

[2]

$$\text{d) } \frac{2abc^2}{6a^2c} = \frac{bc}{3a} \quad \frac{a^m}{a^n} = a^{m-n}$$

[2]

$$\text{e) } (a^2)^5 = a^{10} \quad (a^m)^n = a^{mn}$$

[1]

Page total  $\frac{\quad}{8}$

7) Solve the following equations:

a)  $4x - 2 = -6$

$$4x = -6 + 2$$

$$4x = -4$$

$$x = -1$$

[2]

b)  $3(4 + x) = 21$

$$4 + x = \frac{21}{3} = 7$$

$$x = 7 - 4$$

$$x = 3$$

[2]

c)  $7 - x = 12$

$$7 - 12 = x$$

$$x = -5$$

[2]

d)  $7x - 2 = 25 - 2x$

$$7x + 2x = 25 + 2$$

$$9x = 27$$

$$x = \frac{27}{9}$$

$$x = 3$$

$$x = 3$$

[2]

e)  $\frac{1}{10} - \frac{1}{30} = \frac{1}{x}$

$$\frac{1 \times 3}{10 \times 3} - \frac{1}{30} = \frac{1}{x}$$

$$\frac{3 - 1}{30} = \frac{1}{x}$$

$$2x = 30$$

$$x = 15$$

[3]

Page total  $\frac{\quad}{11}$



8) Write as a decimal the answer to:

$$8 + \frac{8}{10} - \frac{88}{100}$$

$$= 8 + 0.8 - 0.88$$

$$= 8.8 - 0.88$$

$$= 7.92$$

[2]

9) A box in the shape of a cube has a volume of  $8 \text{ cm}^3$

What is the volume of another box whose sides are each twice those of the first one?

$$\text{volume} = 8 \text{ cm}^3$$

$$\text{side}^3 = 8$$

$$\text{side} = 2 \text{ cm}$$

$$\text{New box: side} = 2 \times 2 = 4 \text{ cm}$$

$$\text{volume} = 4^3 = 64 \text{ cm}^3$$

[2]

10) If a car uses 30 litres of fuel when travelling 240 km, how much would it use for 400 km?

30 liters for 240 km

x liters for 400 km

$$x = \frac{30 \times 400}{240}$$

$$240$$

$$x = 50 \text{ liters}$$

[3]

Page total  $\frac{7}{7}$

11) Monty cycles 6 kilometres up a hilly road at a speed of 12 km/h  
 He then cycles 6 kilometres back down at a speed of 24 km/h

a) For how long was he cycling altogether?  
 Give your answer in minutes

$$\begin{aligned} \text{time} &= \frac{\text{distance}}{\text{speed}} = \frac{6}{12} + \frac{6}{24} \\ &= \frac{1}{2} + \frac{1}{4} = \frac{2+1}{4} = \frac{3}{4} \\ &= \frac{3}{4} \times 60 = 45 \text{ minutes} \end{aligned}$$

[2]

b) What was his average speed for the whole journey?  
 Give your answer in kilometres per hour

$$\begin{aligned} \text{average speed} &= \frac{6+6}{45} = \frac{12}{45} \text{ km per minutes} \\ &= \frac{12}{45} \times 60 = 16 \text{ km per hour} \end{aligned}$$

[3]

12) In 9 tests so far this year, Sally has an average mark of 61%.  
 What does she need to score in the 10<sup>th</sup> test to raise her average to 63%?

$$\begin{aligned} \text{total of 9 test} &= 9 \times \frac{61}{100} = 5.49 \\ \text{let score for 10 test be } & \\ \frac{5.49+x}{10} &= \frac{63}{100} \\ 5.49+x &= \frac{63}{10} = 6.3 \\ x &= 6.3 - 5.49 \\ x &= 0.81 = \frac{81}{100} = 81\% \end{aligned}$$

[3]

Page total  $\frac{-}{8}$

13)

a) Find three different integers which add up to +1

$$(-3)+(5)+(-1) = 1$$

Note : other valid answers can be marked as corrected

[1]

b) Find three non-zero whole numbers which add up to +1 and multiply to give +36

$$(-3) + (-2) + 6 = +1$$

$$(-3) \times (-2) \times (6) = +36$$

[2]

14) A bag contains red, blue and yellow counters.

If a counter is picked at random, the probability that it is red is  $\frac{1}{3}$  and the probability that it is blue is  $\frac{1}{4}$

There are 120 red counters in the bag.

a) How many blue ones are there?

let total counters be x

$$\text{Red: } \frac{1}{3} \times x = 120$$

$$x = 360$$

$$\text{blue} = \frac{1}{4} \times 360 = 90 \text{ counters}$$

[2]

b) How many yellow ones are there?

$$\begin{aligned} \text{yellow: } & 360 - (120+90) \\ & = 360 - 210 \\ & = 150 \text{ counters} \end{aligned}$$

[2]

Page total  $\frac{7}{7}$

- 15) In "old money", before British currency was decimalised, there were 12 pence in 1 shilling and 20 shillings in 1 pound.

Work out one-third of a pound in shillings and pence

$$\begin{aligned} 1 \text{ pound} &= 20 \text{ shillings} = 20 \times 12 \text{ pence} \\ \frac{1}{3} \text{ rd pound} &= \frac{20}{3} = 6.67 \text{ shillings} \\ &= \frac{20}{3} \times 12 = 80 \text{ pence} \end{aligned}$$

[3]

- 16) A box has length  $3x \text{ cm}$ , width  $2x \text{ cm}$  and height  $x \text{ cm}$

The total surface area of the box is  $2662 \text{ cm}^2$

- a) Find an expression for the surface area in terms of  $x$  and use it to write down an equation involving  $x$

$$\begin{aligned} \text{Total surface area} &= 2(lw+wh+lh) \\ 2662 &= 2(3x \times 2x + 2x \times x + 3x \times x) \\ 1331 &= 6x^2 + 2x^2 + 3x^2 \\ 1331 &= 11x^2 \\ x &= 121 \end{aligned}$$

[3]

- b) Solve your equation to find the length, width and height of the box.

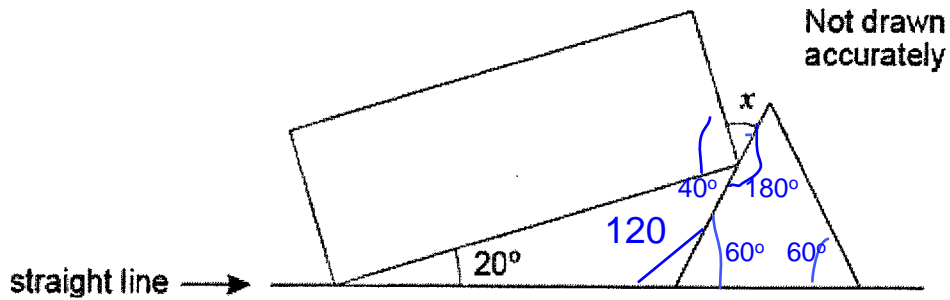
$$\begin{aligned} x &= 11 \text{ cm} \\ \text{length} &= 3 \times 11 \\ &= 33 \text{ cm} \\ \text{width} &= 2 \times 11 \\ &= 22 \text{ cm} \\ \text{Height} &= 11 \text{ cm} \end{aligned}$$

[2]

Page total  $\frac{\quad}{8}$

17)

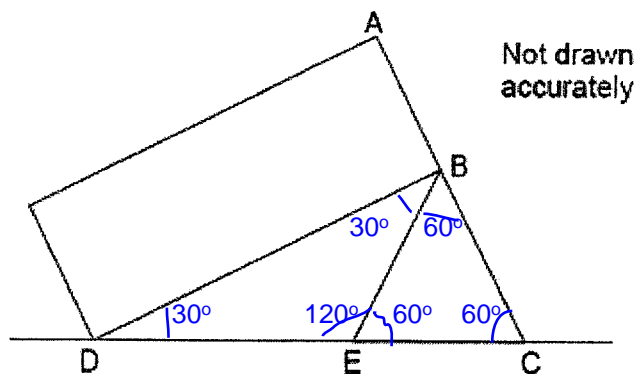
- a) The diagram shows a rectangle which just touches an equilateral triangle. Calculate the angle marked  $x$ . Show your working



At A  $180^\circ - 60^\circ = 120^\circ$  (linear pair)  
 In triangle  $= 180^\circ - (20^\circ + 120^\circ) = 40^\circ$   
 At B,  $90^\circ + x + 40^\circ = 180^\circ$  (linear pair)  
 $x = 180^\circ - 130^\circ$   
 $= 50^\circ$

[3]

- b) Now the rectangle touches the top of the triangle, so that ABC is a straight line. Show that triangle BDE is isosceles.



$\angle DBE = 90^\circ - 60^\circ$   
 $\angle DEB = 180^\circ - 60^\circ = 120^\circ$   
 $\angle BDE = 180^\circ - (120^\circ + 30^\circ)$   
 $= 180^\circ - 150^\circ$   
 $= 30^\circ$   
 $\angle BDE = \angle DBE$   
 $DE = BE$   
 Triangle BDE is isosceles.

[3]

Page total  $\frac{6}{6}$

18) The number 6 has four factors: 1, 2, 3, 6

a) How many factors has the number 49? (List them)

1,7,49

[2]

b) How many factors has the number 16? (List them)

1,2,4,8,16

[2]

c) Find all the numbers between 1 and 100 which have an odd number of factors.  
(There are 10 such numbers altogether)

$1^2 = 1$  (factors :1)

$2^2 = 4$  (factors:1,2,4)

$3^2 = 9$  (factors 1,3,9)

$4^2 = 16$  (factors 1,2,4,8,16)

$5^2 = 25$  ( factors 1,5,25)

$6^2 = 36$  (factors : 1,2,3,4,6,9,12,18,36)

and so on

[3]

d) What do you notice about these numbers?

For perfect square numbers each factor is unique and hence the total factors are odd.

[1]

Page total  $\frac{8}{8}$

19) Find the next three terms and the  $n^{\text{th}}$  term in this sequence:

4, 7, 10, 13, .....

first term  $a = 4$ ,  $d = \text{difference} = 3$

$$\begin{aligned}n^{\text{th}} \text{ term} &= a + d(n-1) \\ &= 4 + 3(n-1) \\ &= 4 + 3n - 3 \\ &= 3n + 1\end{aligned}$$

Next 3 terms are 16, 19, 22

[2]

20)

a) The  $n^{\text{th}}$  term of a sequence is given by the formula  $(n-2)(n+2)$   
Write down the first five terms

$$\begin{aligned}t_n &= (n-2)(n+2) \\ t_1 &= (1-2)(1+2) = -3 \\ t_2 &= (2-2)(2+2) = 0 \\ t_3 &= (3-2)(3+2) = 5 \\ t_4 &= (4-2)(4+2) = 12 \\ t_5 &= (5-2)(5+2) = 21\end{aligned}$$

[3]

b) The  $n^{\text{th}}$  term of another sequence is given by the formula  $n^2 - 4$   
Write down the first five terms

$$\begin{aligned}t_1 &= 1^2 - 4 = -3 \\ t_2 &= 2^2 - 4 = 0 \\ t_3 &= 3^2 - 4 = 5 \\ t_4 &= 4^2 - 4 = 12 \\ t_5 &= 5^2 - 4 = 21\end{aligned}$$

What do you notice about the two sequences?

Both are same.

[3]

c) Use the pattern to work out  $298 \times 302$

$$\begin{aligned}298 \times 302 &= (300 - 2) \times (300 + 2) \\ &= (300)^2 - 4 \\ &= 90000 - 4 \\ &= 89996\end{aligned}$$

[2]

Page total  $\frac{\quad}{10}$

21) Purple Fire paint is made by mixing red and blue paint in the ratio 3:1.

Midnight Glow is made by mixing red and blue paint in the ratio 1:3.

By accident, 1 litre of Midnight Glow is mixed into 2 litres of Purple Fire.

a) How much red and how much blue paint is in the resulting mixture? (You might find it easier to work in millilitres)

$$\text{purple fire: Red} = \frac{3}{4} \text{ and blue} = \frac{1}{4}$$

$$\text{Midnight glow} = \text{Red} = \frac{1}{4} \text{ and blue} = \frac{3}{4}$$

$$\text{total blue} = \frac{3}{4} \times 1 \text{ liter} + \frac{1}{4} \times 2 \text{ liter} = \frac{5}{4} \text{ liter} = \frac{5}{4} \times 1000 = 1250 \text{ ml}$$

$$\text{Total red} = \frac{1}{4} \times 1 \text{ liter} + \frac{3}{4} \times 2 \text{ liters} = \frac{7}{4} \text{ liters}$$

$$\frac{7}{4} \times 1000 = 1750 \text{ ml}$$

[3]

b) How much paint, and of which colour, needs to be added to convert the resulting mixture back to Purple Fire?

$$\text{Total mixture} = 1250 + 1750 = 3000 \text{ ml}$$

Ratio of red and blue paint for purple fire = 3:1

$$\text{Red should be } \frac{3}{4} \times 3000 = 2250$$

$$\text{Blue should be } \frac{1}{4} \times 3000 = 750$$

1000 ml of red paint is required to be added.

[3]

Page total  $\frac{-}{6}$

**End of questions**

**Maximum mark 100**